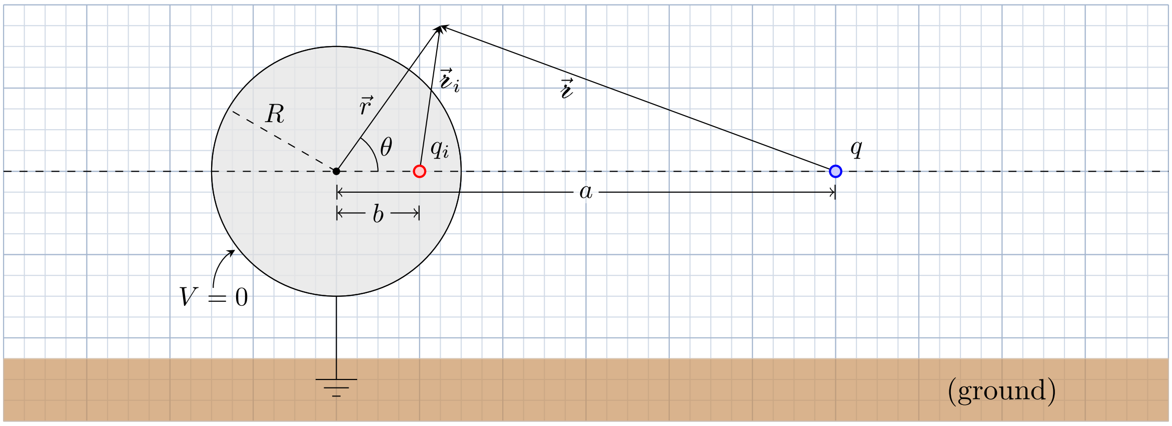
Problem 1: Point charge and a conducting sphere

In Example 3.2, Griffiths uses the method of images to find the potential due to a point charge a distance a from the center of a grounded, conducting sphere.



(a) Following his example (pages 128-129 in the 4^{th} edition; pages 124-125 in the 3^{rd} edition), derive the following result for the potential in the region outside the sphere

$$V(r,\theta) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{r^2 + a^2 - 2\,ra\cos\theta}} - \frac{q}{\sqrt{R^2 + \left(\frac{ra}{R}\right)^2 - 2\,ra\cos\theta}}\right].$$

(b) Find the induced surface charge on the sphere as a function of the polar angle θ. Integrate this charge density over the sphere to determine the total induced charge. (Even if the sphere starts off electrically neutral, it acquires charge as the point charge approaches it. It can do this because it is grounded, so charge can flow up from or down into the ground as needed.)

(c) Calculate the energy of this configuration. (HINT: Find the force on the charge q, then integrate $-d\vec{\ell} \cdot \vec{F}$ from infinitely far away to a point at distance a from the center of the sphere.)

Problem 2: Point charge and a conducting sphere

In the previous problem the conducting sphere was grounded, so that $V = 0$ throughout. In that case only a single image charge was needed to find the potential outside the sphere. What about a conducting sphere that is not grounded, so that the potential is some constant V_0 throughout? You can accomplish this by adding a second image charge q_{i2} to your answer from the last question.

(a) Where should the second image charge q_{i2} be located?

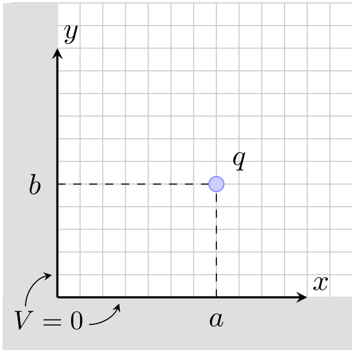
(b) What is the resulting expression for the potential in the region outside the sphere?

(c) Suppose the conducting sphere is electrically *neutral*. Then its total charge, which also must be the same as $q_i + q_{i2}$, is zero. What is the potential V_0 throughout the conductor when the charge q is a distance a from the center of the sphere?

(d) Find the force that the neutral conducting sphere exerts on the charge q.

Problem 3: Point charge near grounded conducting planes

Two semi-infinite grounded conducting planes (the shaded portions of the figure) meet at a right angle. In the region between them there is a point charge q. Set up the image configuration, and calculate the potential in this region.



(a) What charges do you need, and where should they be located?

(b) What is the force on q?

(c) How much work did it take to bring q in from infinity?

Problem 4: Point charge moving towards a conducting plane

A point charge q of mass m is released from rest at a distance d from an infinite grounded conducting plane. Show that the charge hits the plane after an amount of time given by

$$\Delta t = \frac{\pi d}{q}\sqrt{2\pi\epsilon_0 m d}\ .$$

NOTE: This one is a bit tricky, so let me give you a few pointers for solving this type of problem. Start by finding the force experienced by the charge as a function of z, and set this equal to $m\,\mathrm{d}^2z/\mathrm{d}t^2$ (the mass times the acceleration). This gives a differential equation that is hard to solve directly. We can get around this by multiplying both sides by $\mathrm{d}z/\mathrm{d}t$; this will allow us to write each side of this new equation as a total derivative. Since each side is now a total derivative you can easily integrate, which gives you an equation that relates $\mathrm{d}z/\mathrm{d}t$ to z. Now, since you integrated the last equation your new equation should have an unspecified constant on one side. Fix this constant by setting an *initial condition*. Specifically, we're interested in solutions of this new equation where the charge starts from rest when it is a distance d from the plane, so figure out what the constant needs to be so that $\mathrm{d}z/\mathrm{d}t = 0$ when $z = d$. (You've just implemented conservation of energy, starting from Newton's 2^{nd} law with a conservative force.) Once you've fixed the constant in your equation, a little algebra should let you rewrite it in the form

$$\mathrm{d}t = \mathrm{d}z\ \times \text{(a function that depends on } z,\, d,\, m, \text{ etc...)}$$

Now integrate both sides of this equation to get the answer. What should the limits of the integral on the right-hand-side be? Does your final answer make sense if you keep the charge q fixed but make the mass m larger or smaller? What if you fix m but let q become larger or smaller?